Midterm Exam Practise Paper

1 Solve the inequality $2 - 3x < |x - 3|$. [4]

2 The polynomial $2x^3 + ax^2 - 4$ is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

When a has this value,

(ii) factorise $p(x)$, [2]

(iii) solve the inequality $p(x) > 0$, justifying your answer. [2]

3 When $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

(i) Find the value of a. [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

4 **(i)** Express $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in ascending powers of x, up to and including the term in x^2. [5]

5 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

6 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $p(x)$.

(i) Find the value of a and hence factorise $p(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\dfrac{8x - 13}{p(x)}$ in partial fractions. [5]

7 Expand $\dfrac{1}{(2 + x)^3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

8 Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

9 The variables x and y satisfy the equation $y^3 = Ae^{2x}$, where A is a constant. The graph of $\ln y$ against x is a straight line.

(i) Find the gradient of this line. [2]

(ii) Given that the line intersects the axis of $\ln y$ at the point where $\ln y = 0.5$, find the value of A correct to 2 decimal places. [2]

10 Solve the equation $\ln(2 + e^{-x}) = 2$, giving your answer correct to 2 decimal places. [4]

11 It is given that $\tan 3x = k \tan x$, where k is a constant and $\tan x \neq 0$.

(i) By first expanding $\tan(2x + x)$, show that

$$(3k - 1)\tan^2 x = k - 3.$$ [4]

(ii) Hence solve the equation $\tan 3x = k \tan x$ when $k = 4$, giving all solutions in the interval $0° < x < 180°$. [3]

(iii) Show that the equation $\tan 3x = k \tan x$ has no root in the interval $0° < x < 180°$ when $k = 2$. [1]

12 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \tfrac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

13 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

14 Prove the identity

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x.$$ [3]

15 Solve the equation

$$\cos\theta + 3\cos 2\theta = 2,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

16 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]